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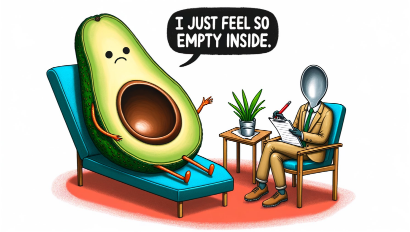

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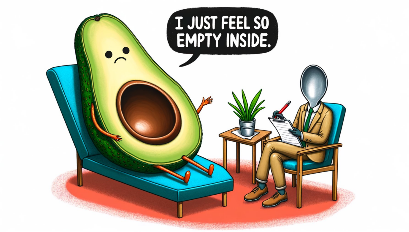

- Have a complicated distribution (say over images) q_0, would like to *learn* the distribution and then *sample* from it

Forward and Reverse SDE

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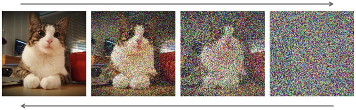

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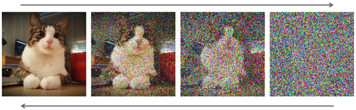

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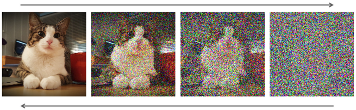

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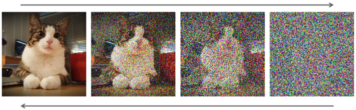

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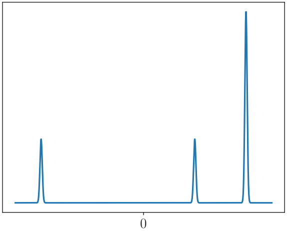

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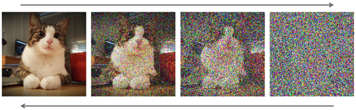

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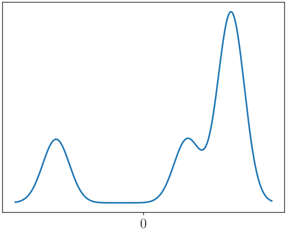

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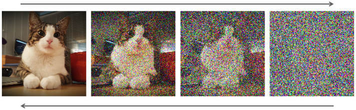

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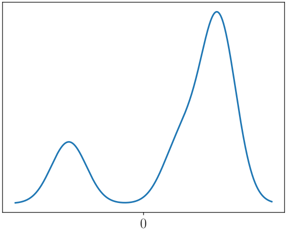

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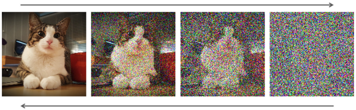

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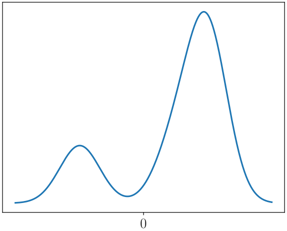

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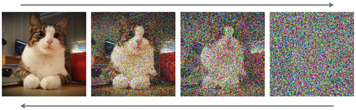

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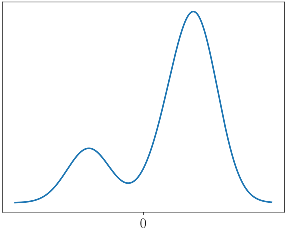

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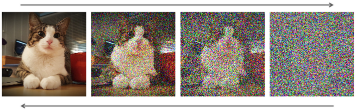

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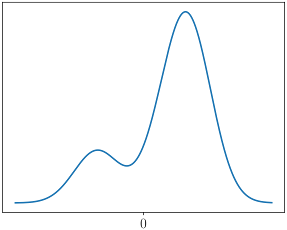

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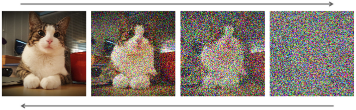

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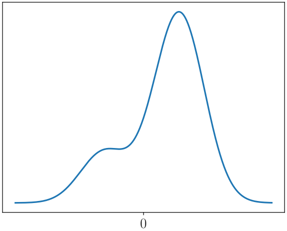

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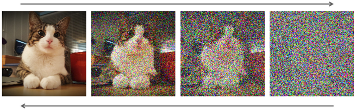

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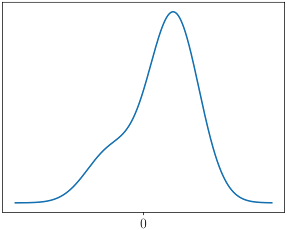

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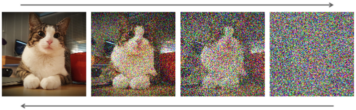

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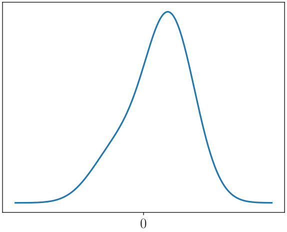

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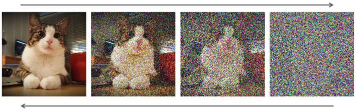

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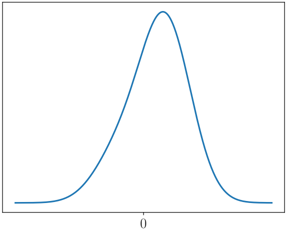

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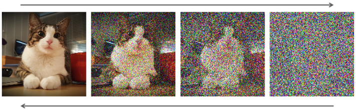

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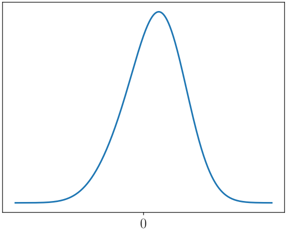

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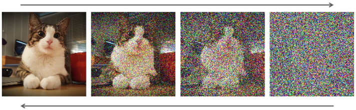

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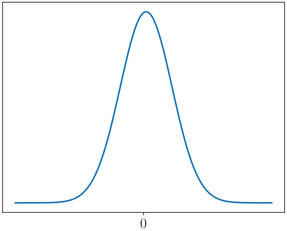

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- Remarkably, both these quantities have good theoretical bounds

Prior Work

- For γ-Wasserstein error, ε-TV error, and hypothesis class $\mathcal{H}$,

Samples to Train	**Steps to Sample**
poly $\left(d, \frac{1}{\varepsilon}, \frac{1}{\gamma}, \log \lvert\mathcal{H}\rvert\right)$ [BMR20][1]	poly $\left(d, \frac{1}{\gamma}, \frac{1}{\varepsilon}\right)$ [CCL+22][2]

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Can we **train** using a number of **samples** scaling polylogarithmically in $\frac{1}{\gamma}$ just like the number of steps?

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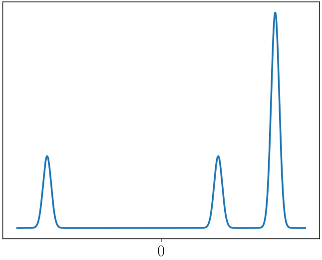
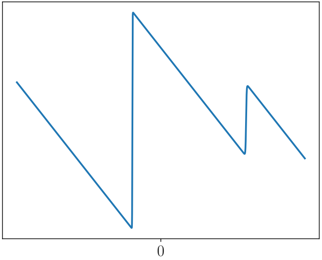
$\widetilde{O}\left(\frac{d^2}{\varepsilon^5}\log^3\frac{1}{\gamma}\log\lvert\mathcal{H}\rvert\right)$ **[Ours]**	$\widetilde{O}\left(\frac{d\log^2\frac{1}{\gamma}}{\varepsilon^2}\right)$ [BBDD23]

Only require second moment to be between $\frac{1}{\mathrm{poly}(d)}$ and $\mathrm{poly}(d)$.

Intuition





- To get $\operatorname{poly}\log\frac{1}{\gamma}$ dependence for **sampling**, [CLL23][4], [BBDD23] observe that score function better behaved with increasing noise. Can tolerate larger score error for small t, proportional to $\frac{1}{\min(1,t)}$.

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*In a weaker sense than L^2, but sufficient for sampling

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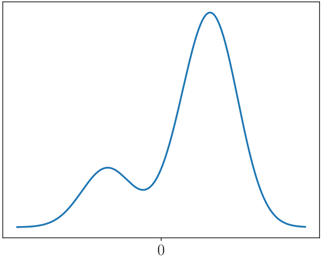

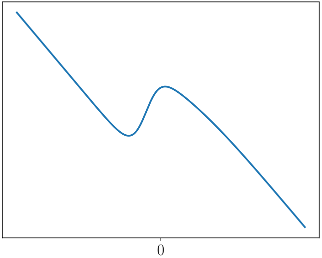

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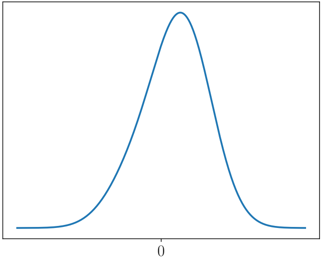

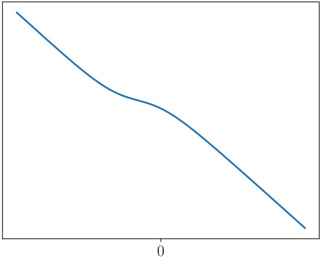

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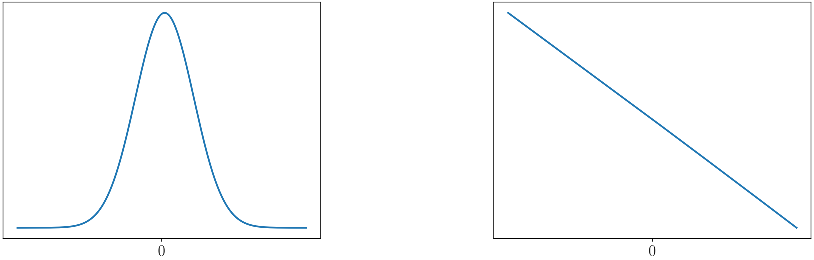

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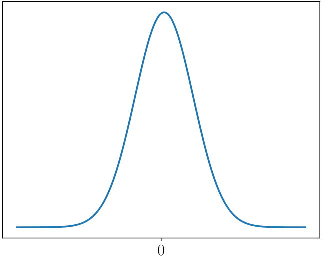

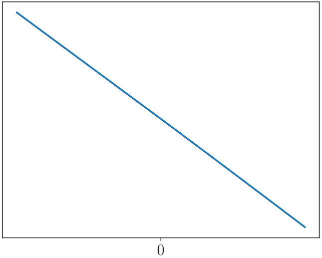

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- We exploit this for **training** – show $\text{poly}\log\frac{1}{\gamma}$ dependence for sample complexity to learn score*.

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Conclusion

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Samples to Train	**Steps to Sample**
poly $\left(d, \frac{1}{\varepsilon}, \frac{1}{\gamma}, \log \lvert\mathcal{H}\rvert\right)$ [BMR20]	poly $\left(d, \frac{1}{\gamma}, \frac{1}{\varepsilon}\right)$ [CCL+22]
$\widetilde{O}\left(\frac{d^2}{\varepsilon^5} \log^3 \frac{1}{\gamma} \log \lvert\mathcal{H}\rvert\right)$ **[Ours]**	$\widetilde{O}\left(\frac{d \log^2 \frac{1}{\gamma}}{\varepsilon^2}\right)$ [BBDD23]